Magnetization Currents

Recall that the magnetic vector potential $\mathbf{A}(\bar{r})$ created by **volume** current distribution $\mathbf{J}(\bar{r})$ is:

$$\mathbf{A}(\bar{\mathbf{r}}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\bar{\mathbf{r}}')}{|\bar{\mathbf{r}} - \bar{\mathbf{r}}'|} dv'$$

while the magnetic vector potential created by a **surface** current $\mathbf{J}_s(\bar{r})$:

$$\mathbf{A}(\bar{\mathbf{r}}) = \frac{\mu_0}{4\pi} \iint_S \frac{\mathbf{J}_s(\bar{\mathbf{r}}')}{|\bar{\mathbf{r}} - \bar{\mathbf{r}}'|} ds'$$

Therefore, if **both** volume and surface current densities are present we find that the **total** magnetic vector potential is:

$$\mathbf{A}(\bar{\mathbf{r}}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\bar{\mathbf{r}}')}{|\bar{\mathbf{r}} - \bar{\mathbf{r}}'|} dv' + \frac{\mu_0}{4\pi} \iint_S \frac{\mathbf{J}_s(\bar{\mathbf{r}}')}{|\bar{\mathbf{r}} - \bar{\mathbf{r}}'|} ds'$$

Compare these expressions to the magnetic vector potential field produced by material with **Magnetization Vector** $\mathbf{M}(\bar{r})$:

$$\mathbf{A}(\bar{r}) = \iiint_V \frac{\mu_0 \mathbf{M}(\bar{r}') \times (\bar{\mathbf{r}} - \bar{\mathbf{r}}')}{4\pi |\bar{\mathbf{r}} - \bar{\mathbf{r}}'|^3} dv'$$

We can write also write this expression as (trust me!):

$$\mathbf{A}(\bar{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\nabla' \times \mathbf{M}(\bar{r}')}{|\bar{r} - \bar{r}'|} dv' + \frac{\mu_0}{4\pi} \oiint_S \frac{\mathbf{M}(\bar{r}') \times \hat{a}_n}{|\bar{r} - \bar{r}'|} ds'$$

where surface S is the **closed surface** that surrounds material volume V, and unit vector $\hat{a}_n$ is **normal** to this surface.

We find that this is identical to the expression:

$$\mathbf{A}(\bar{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\mathbf{J}(\bar{r}')}{|\bar{r} - \bar{r}'|} dv' + \frac{\mu_0}{4\pi} \iint_S \frac{\mathbf{J}_s(\bar{r}')}{|\bar{r} - \bar{r}'|} ds'$$

if $\mathbf{J}(\bar{r}) = \nabla \times \mathbf{M}(\bar{r})$ and $\mathbf{J}_s(\bar{r}) = \mathbf{M}(\bar{r}) \times \hat{a}_n$.

Therefore, we find that the magnetization of some material, as described by magnetization vector $\mathbf{M}(\bar{r})$, creates **effective** currents $\mathbf{J}_m(\bar{r})$ and $\mathbf{J}_{sm}(\bar{r}_s)$ (where $\bar{r}_s$ indicates points on the material **surface**). We call these effective currents **magnetization currents:**

$$\mathbf{J}_m(\bar{r}) = \nabla \times \mathbf{M}(\bar{r}) \qquad \left[\frac{A}{m^2}\right]$$

$$\mathbf{J}_{sm}(\bar{r}_s) = \mathbf{M}(\bar{r}_s) \times \hat{a}_n \qquad \left[\frac{A}{m}\right]$$

Again, note the **analogy** of these **magnetization** currents with **polarization** charges $\rho_{vp}(\bar{r})$ and $\rho_{sp}(\bar{r})$.